Optimization in Component Design

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The process of optimization is at the core of every organic or synthetic design. Although sometimes implicit, optimization manifests itself in every step of the creative process from finding spatial fit to achieving cost effective delivery methods. The process is most applicable in the field of engineering design, where the multi-dimensional and multi-disciplinary aspect of the practice creates a vast opportunity for finding the best solution. This paper discusses the generalized formulation of structural optimization, and presents some relevant techniques for adaptation in curtainwall products. Specifically it discusses two approaches of "topology" and "shape" optimization and the potential of their application in the design of components in unitized curtainwall systems. Topology optimization is the means of creating material distribution for a given set of loads and boundary constraints. The method of Solid Isotropic Material and Penalization (SIMP) is employed to demonstrate the features of this technique in design of a typical curtainwall anchorage assembly. Shape optimization is a method for crafting the most efficient geometry to meet a set of specified conditions, while minimizing the area of the resulting geometry. To investigate the possibilities of shape optimization, the "Evolutionary Solver Method" is used, and the feasibility of this methodology is demonstrated by searching for best possible extrusion profile for typical unitized curtainwall mullions.

> Over **two million pounds of aluminum** are used in the fabrication of the curtainwall system for a **typical 45-story building**.

1 INTRODUCTION

All engineering designs have the goal of achieving the best solution to a “set of variables” constrained by a “set of limitations”. This challenge is exacerbated when combined with ever increasing demands for competitiveness and the need for superior performance. Curtainwall designs follow the same requisite. Here the optimum solution is the safest, the most efficient, and the most economical product that satisfies a number of physical, spatial, and aesthetic criteria. Consider the following observations:

- Over two million pounds of aluminum are used in the fabrication of the curtainwall system for a typical 45-story building. This material costs approximately 8 to 9 million U.S. dollars.
- The annual aluminum market for the construction industry in the United States is approximately 350 to 400 million pounds. This accounts for about 13 to 15 percent of total aluminum usage in the U.S.
- Aluminum has the highest energy production ratio (44,711 Btu/lb) of all of the material used in the construction industry (glass is a close second). In comparison steel uses only 8,700 Btu per pound of steel.
- Approximately 30 percent of the price of the aluminum is the cost of the electricity used to produce it.

These statistics underline the assertion that it is both economically prudent and an environmental responsibility to aim for high levels of efficiency in the design of aluminum products. As such, the practice of reducing the product weight while maintaining structural integrity is paramount to the design of good aluminum extrusions and castings used in curtainwall systems.

In general, the following three steps are followed in the design of system components:

1. Function: This task determines the utility of the product - for example, vertical mullions are used to frame the glass and to transfer distributed lateral loads to the floor anchors and be of a sufficient stiffness to avoid excessive movement. The anchors transfer this load to the support slab and provide the required means to accommodate building movement. Primary mechanics of the element dictate the general schematics of the component. At this step the material is chosen, though the material property remains to be further examined.

2. Conceptual Design: At this stage, the design will be evaluated for fit and spatial consideration. If the parts are visual, the aesthetic aspects are evaluated in conjunction with the performance requirements. Interface with other mating components will be detailed. The result is a design with a set of specifications and constraints. In the example of vertical mullions, factors such as outer profile, unit dimensions, finish types, and thermal isolation will be established.

3. Optimization: The final step is to find the most efficient form of the component. Here several elements, ranging from strength to economy and quality to quantity, will be examined in search of the best design.

This last step is the topic of this report. Traditionally the optimization process has been an intuitive and iterative course, where the design starts with experiential or perceptive concepts, and then a series of refinements are made to converge to a more effective and robust design. However, this approach becomes less efficient as the number of variables increase and design goals become multi-objective. For example, design for both stress compliance and displacement rigidity require interrelated analysis, which is not only subject to independent specifications but also sometimes contradictory.

Let’s consider the design of a simple beam subjected to a uniformly distributed load (see figure 1). To attain lower magnitudes of vertical displacement, it is instinctive to increase the depth of the beam. But as the depth increases the cross-section becomes more susceptible to twist, hence less stable, requiring a higher torsional rigidity. This problem becomes more tedious when the number of parameters in the definition of cross sectional geometry increases (for example variables such as wall thicknesses, open and multi-cell profiles).

The following sections present a brief summary of the optimization history and its role in modern engineering practices. General formulation and representative examples highlighting the application of structural optimization in curtainwall design conclude the report.

2 HISTORY

The earliest account of an optimization process is from nearly 3000 years ago. The legend as explained by Virgil has it that Queen Dydo’s fled her native Greece from the fear of being killed by her brother and landed on the shores of North Africa (present day Tunisia). Upon arrival she asked the natives to purchase a piece of land to settle. The local chieftain offered her as much land as she could enclose within a hide of a bull. The Queen accepted the proposition and proceeded to cut the hide to small strips and tie them together to make a long string. Then she laid the string in a semi-circle arc with Mediterranean shore as the straight boundary. This way she created the largest area for the city known as Carthage. The problem Queen Dydo solved, covering the largest area with the smallest perimeter, is today known as isoperimetric problem and is still a topic of study in the field of calculus of variations.

The formal solution to the mathematical problem of optimization was first introduced in early 19th century by German mathematician Gauss and leaped into an applied science in early twentieth century. Since then, thousands of problems, in hundreds of fields from economics and social sciences to engineering and genetics have been classified in one form or other as an optimization problem. The advent of numerical computation has boosted the practice to a widely acceptable means of solving complex physical and mathematical problems. Escalating needs for higher speeds and smaller sizes have made optimization the cornerstone for all disciplines in applied science.

In the field of structural mechanics, the concept has been an active area of research for the past fifty years. Methodical optimization in applied mechanics has become significant over the past two decades. A recent collection of papers by Arora [2] is an excellent source of the state of development in the subject. Other text by Christensen and Klarbring [4], Haung and Xie [5] and Bensoe and Sigmund [3] give an in-depth discussion of structural optimization techniques. In practice, most of the comprehensive solid mechanics software packages offer some type of optimization module, which can be applied to a variety of computational models.

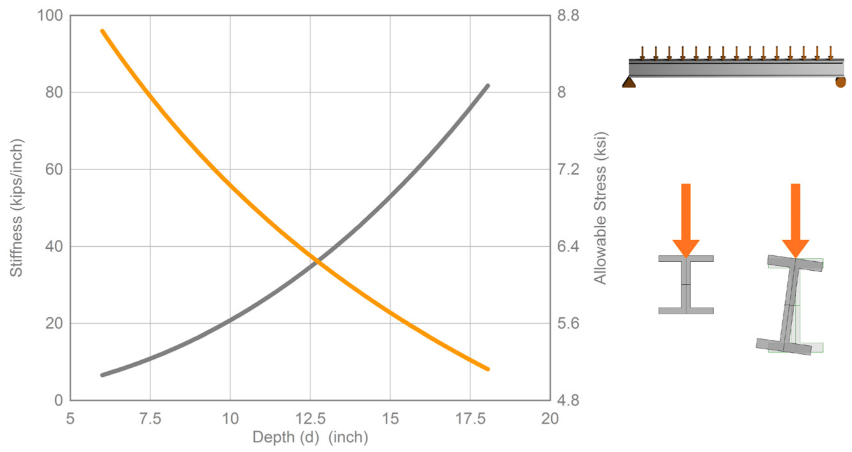


Figure 1: Effect of depth change on the performance of a simple supported beam. As depth increases the stiffness increases, and the deflection reduces. At the same time by incasing the depth, the potential for twisting of the bean increases, hence reduced allowable stresses (orange curve).

3 OPTIMIZATION PROBLEM

All optimization problems in engineering design are formulated using the following four elements:

- Objective Function - J(): This is a function describing the measure of the goodness of design. Sometimes referred to as cost function, it is a single or multiple objective that needs to be minimized (or maximized). It usually refers to product weight, spatial coordinates, strain energy, material cost, or a combination of these attributes.

- Design Variables - {Vd} : These are a set of variables that form the design. They can be geometrical (such as shape or thickness) or physical (such as material strength or density).

- State Variables - {Vs}: These variables are not at the designer control; however, they will have effect on the feasibility of the final product and/or the value of the objective function. Examples might include induced stress or deformations.

- Constraints - G(): These are the limitations imposed on the design. They may be the practical range of the design variables, the behavioral restraints of the state variables or the laws governing the physics of the problem.

The problem then becomes finding the best set of variables Vd so the function J(Vd,Vs) is minimized (maximized) subject to a group of constraints imposed by functions {G(Vd,Vs)}. This framework, sometimes referred to as a mathematical programming problem (no relation to computer programming), is the simplest form of the optimization problem.

When it comes to finding a solution, the optimization problem is somewhat deceiving in its simplicity. Consider the case of a simple example depicted in figure 1. Here we have a set of two design variables {v1, v2}. The Objective function J() is surface generated by the admissible values of the design variables in their respective bounds. As it can be seen, any solution system needs to deal with a highly random nature of the objective function to reach the peak while trying to avoid any local peaks. If the cost function is definite and rational, it is possible to find a closed form solution. But in general, the cost functions are highly non-linear in nature and require advanced numerical algorithms. Furthermore, the number of variables in a problem could range in tens if not hundreds, making any form of a simplified solution insensitive to the design parameters. The study of solution techniques for solving optimization problems is outside the interest of this paper. When discussing the case study examples, a brief explanation of the solution technique will be presented.

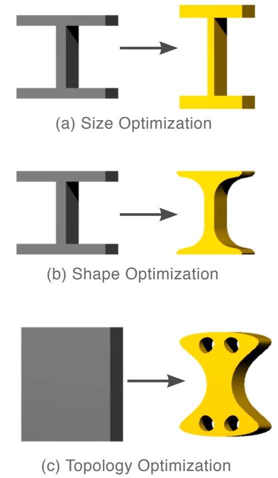


Figure 2: Types of component optimization; (a) Size Optimization: width, depth and thicknesses of flanges, and web are the design variables, (b) Shape Optimization: the outer profile of the geometry is the design variable, (c) Topology Optimization: the material distribution inside the cross section is the design variable.

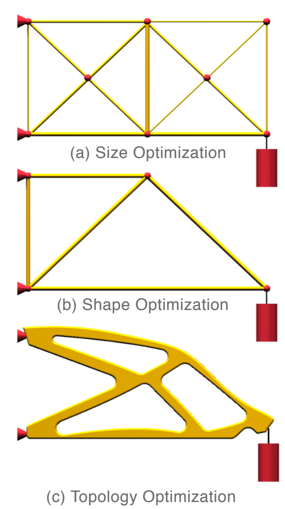


Figure 3: Types of structural optimization; (a) Size Optimization: tube diameter and wall thickness of the struts are the design variables, (b) Shape Optimization: the proper placement of the struts define the shape of the truss, (c) Topology Optimization: the optimum material placement constructs the optimal skeleton of the truss.

Figure 4: Typical unitized curtainwall anchorage assembly (left), and a custom anchor at the L.A. Live Conference Center in Los Angeles (right).

4 STRUCTURAL OPTIMIZATION

Structural optimization is the process involving the search for geometry to achieve the highest performance employing the least amount of material. In general the goal is to create the best design given a set of support conditions and applied loads. Prevailing literature classifies the structural optimization in three categories: *size*, *shape*, and *topology*. This categorization may be applied to the designs both at the component level (micro), or structural level (macro). This distinction is demonstrated in figures 2 and 3.

Size optimization assigns parametric data to a given geometry. In this process the overall shape and topology of the design remains constant while mollifying dimensional information. In the case of components, these could be profile thicknesses, depth and height. For structural optimization, the process usually entails the selection of the most efficient cross section from a table of allowable properties. Material properties such as strength and stiffness can also be some of the parameters being optimized.

The *shape optimization* approach aims to find the most efficient design by defining the boundary contour of the product or structure. The basic premise of the method is to represent the confines of the model with a series of curves, which can be changed to attain a superior distribution of forces. In the case of component design, size optimization could be treated as a subset of shape optimization. However, in practice, there are usually visual or functional constraints that predefine the overall profile, whereupon sizing becomes the predominant means of optimization.

Topology optimization is the progression of the previous methods into a general technique of structural optimization. This method is a process of laying material within a given space, which results in the most effective design. In component design, the outcome might be profiles with voids or hollows; in structural design, the result will be a demonstration of load paths through the model. This scheme is the most complex scenario to implement. However, it is the most robust with the most efficient solution.

Applying all or some of these techniques may optimize curtainwall systems. Different parts tend to match unique approaches for obtaining the right solution. For example, shapes using extrusion processes benefit more from shape optimization, and components made using casting or machining solid blocks tend to be better suited to topology optimization.

5 ANCHOR HOOK DESIGN: EX. 1

To demonstrate the utility of the optimization process, let's consider the example of a typical anchor system used to attach unitized curtainwall system to building floor, in particular the part that engages into mullions and hooks to the anchor assembly attached to the slab (see figure 4). This component transfers gravity loads as well as lateral horizontal loads to support structure while accommodating movements due to thermal effects, live loads, and in plane seismic and wind loads. This element is common to almost all unitized systems in a variety of configurations. The fabrication methods include machined aluminum extrusions, cast or forged aluminum or steel.

The objective of this optimization process is to design the most effective anchor system that minimizes the amount of the material used while maintaining full structural integrity and load transfer mechanisms of the element. To this end, a topology optimization technique known as Solid Isotropic Material with Penalization or SIMP is introduced here.

The basic premise of SIMP is to distribute material mass to achieve an efficient stress distribution within the part. One approach would be to assume that the perimeter of the component is to remain constant, while the thickness of material is varied to obtain the optimum topology.

Computationally this scenario can be implemented by defining the concept of "effective" modulus of elasticity, which is defined by the following relation:

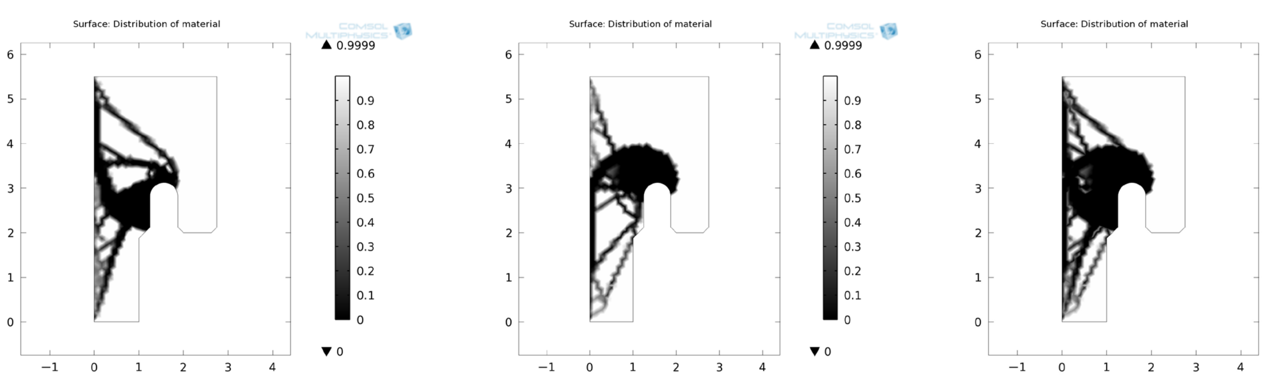
$$E_e(x,y) = \rho^q(x,y) \cdot E$$

In this equation (E_e) is the effective spatial modulus of the elasticity, (ρ) is the distributed material density ranging between 0 and 1, and (q) is an integer constant greater than one (usually assigned a value of 3).

Substituting this equation into the formulation of minimizing strain energy principals, a non-linear finite element analysis can be formulated to be solved iteratively. The implementation details are discussed by Christensen and Klarbring [4] and Haung and Xie [5].

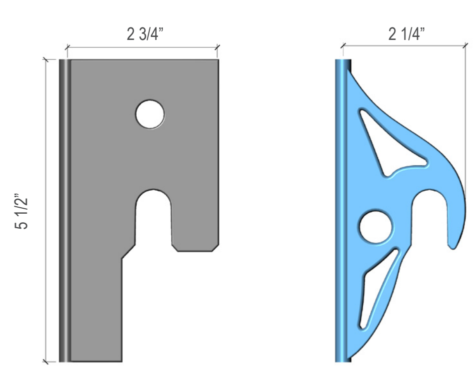
This methodology is applied to the anchor hook. Figure 5 depicts the mass distribution in a typical hook design for both negative and positive applied lateral loads combined with the gravity load. The mass density is then depicted in grayscale images, after which these images are combined graphically into a single image that can be used as a template for a new design (see figure 6).

The stress analysis of this new design is compared to the original element in figure 7. It can be seen from the results that the new design has a more efficient distribution of stresses while reducing the weight by 60 percent. Table 1 compares the statistical average and peak stresses of the two solutions, indicating a dramatic distinction between the two designs.



Pressure Suction Superimposed

Figure 5: Results of SIMP mass distribution of typical anchors.



Design A Design B

Figure 6: Design A is the original anchor profile, manufactured by machining a longitudinal extrusion. The final product weighs 3.2 pounds. Design B is the revised shape using SIMP topology optimization. The part weighs 1.4 pounds, a reduction of 56 percent.

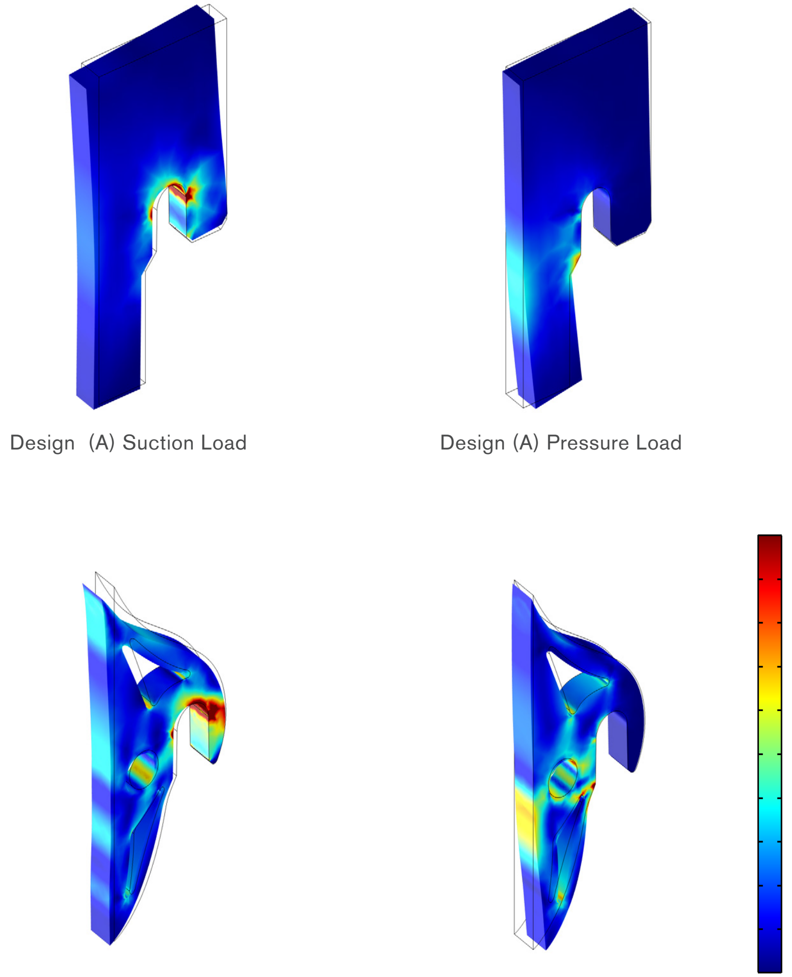


Figure 7: Finite element analysis of a typical anchor A, and the optimized version for pressure and suction loads. Von Mises stress distribution.

Loading Type	Evaluation Metric	Design A Weight = 3.2 lb	Design B Weight = 1.4 lb
Pressure (+)	$(s_{vm})_{max}/s_y$	0.36	0.67
	$\int_V s_{vm}\, dv$	328 ft-lb	361 ft-lb
Suction (-)	$(s_{vm})_{max}/s_y$	1.09	0.68
	$\int_V s_{vm}\, dv$	472 ft-lb	438 ft-lb

Table 1: Comparison of average and maximum stresses between a standard anchor (Design A) and the optimized version (Design B) for pressure and suction wind loads.

6 VERTICAL MULLION DESIGN: EX. 2

Consider a vertical aluminum mullion of a typical span, where the target cross-sectional profile is to follow an outside shape as depicted in figure 8. It is the objective of the optimization process to achieve the highest capacity with lowest material weight (smallest cross-sectional area). We can define the efficiency index as the load carrying capacity of the mullion per unit length divided by its weight per unit length. This index is defined as our cost function to be minimized. The optimum design is considered to be a function of two primary state variables, center span deflection ($\delta < L/175$); and the maximum allowable stress in the compression fiber of the cross section (F_c). The design variables are the various wall thicknesses of the extrusions, which define section properties such as moment of inertia, section modulus and torsional constants. The constraints are defined by the following equations:

$$\delta = \frac{5wL^4}{384EI_{xx}}$$

$$\delta = h(I_{xx}^{-1})$$

$$\alpha = \frac{L(I_{xx}/d_c)}{C_b\sqrt{I_{yy}J/2}}$$

$$F_c = \begin{cases} \dfrac{F_{cy}}{n_y} & \alpha \le S_1 \\ \dfrac{1}{n_y}\left(B_c - 1.6D_c\sqrt{\alpha}\right) & S_1 < \alpha < S_2 \\ \dfrac{\pi^2 E}{2.56 n_y \alpha} & S_2 \le \alpha \end{cases}$$

The detailed description of the various variables defined in the above equations is available in the Aluminum Design Manual [1]. The answer to this optimization problem is obtained using an evolutionary solution technique, common to problems with multiple variables and relatively simple cost evaluation schemas. Evolutionary algorithms perform well in approximating solutions to a variety of optimization problems when there are weak postulations of underlying fitness of design variables, and the solution can converge into multiple possibilities that can be manually redirected. Apart from their use as mathematical optimizers, evolutionary computation and algorithms have also been used as an experimental framework within which to validate theories in biological evolution and natural selection. The mathematics of this solution technique is beyond the scope of this paper; however, there are numerous publications exploring the fundamentals and the implementation techniques of the concept.

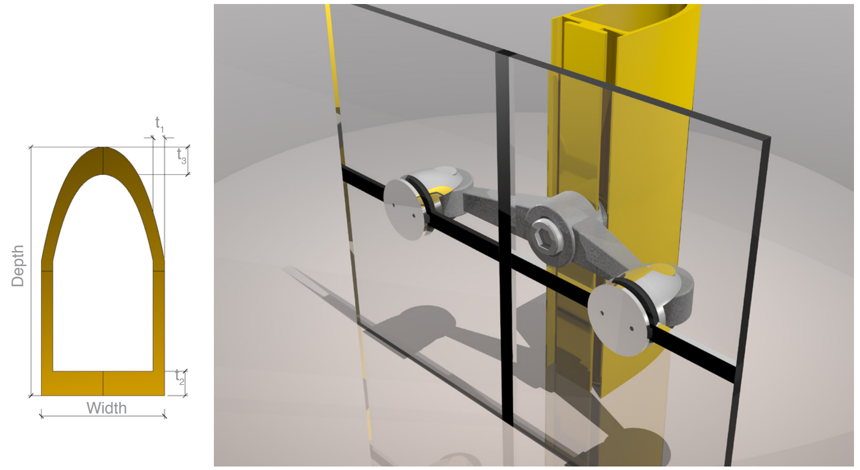


Figure 8: Shaped vertical mullion system and its parametric description.

Capacity (plf)	250	200	150
Depth (in)	6	6	6
Width (in)	3	3	3
t_1 (in)	1/8	3/16	1/8
t_2 (in)	7/32	1/4	5/32
t_3 (in)	1/2	9/32	1/8
Wt (plf)	3.48	2.66	2.33
Efficiency Index	73	77	62

Table 2: Cross section dimensions and mullion properties for optimized profiles, for different applied loads.

Parameter	Design (A)	Design (B)
d_0 (in)	6.00	6.00
d_1 (in)	1.19	1.00
d_2 (in)	0.75	1.44
w_1 (in)	2.00	1.50
t_1 (in)	1/8	3/16
t_2 (in)	1/8	3/32
t_3 (in)	1/8	1/16
t_4 (in)	1/8	7/32
t_5 (in)	1/8	3/32
t_6 (in)	1/8	1/4
Area (in^2)	1.81	1.49
I_{xx} (in^4)	6.95	6.97
I_{yy} (in^4)	0.97	0.42
J (in^4)	0.40	1.56
S_C (in^3)	1.87	1.85
w_d (plf)	123.6	123.6
w_f (plf)	166.8	152.4
Efficiency Index	58	70

Table 3: Cross section dimensions of split vertical mullion and efficiency index for Standard design A and an optimized version B.

Figure 9 (opposite): Typical split vertical mullion (blue). From left to right: actual mullion; parametric description; original design; optimized design.

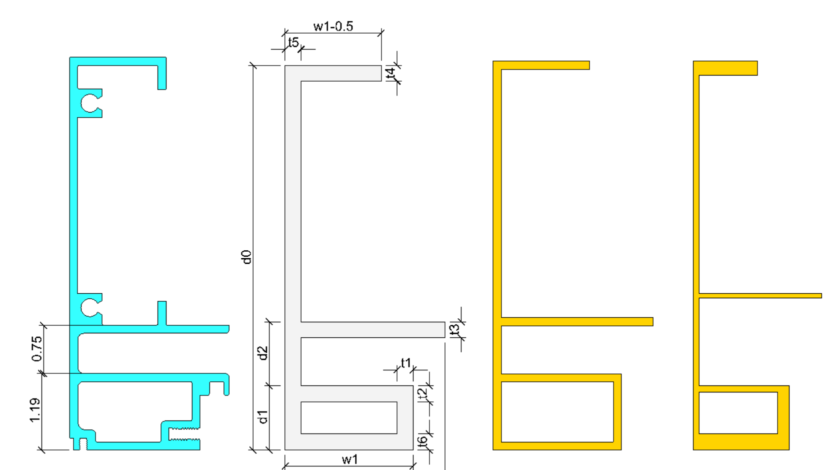
Table 2 shows the results for three different applied load conditions, and the corresponding optimized parameters. The wall thicknesses are constrained to remain within a 1/8" – 1/4" dimension, with a resolution of 1/32". It is interesting to note than that the efficiency index is reduced for the lower applied loads (150 pounds per linear feet). This indicated that the sensitivity of the design parameters are reduced at this load level. To gain a more efficient solution we need to consider adding additional design variables such as profile depth and width.

A similar process is applied to a split mullion of a unitized curtain wall system. Figure 9 shows the split mullion and a simplified model with a selection of design variables. The state variables are again the mid span deflection limitation (span divided by 175) and maximum allowable stress in the mullion. Table 3 presents the properties of the original designs as they compare to the optimized system. As expected, the optimization process moved mass away from the center of the mullion and placed it in the extreme fibers. The resulting design attained a 21 percent increase in the efficiency index of the cross section while maintaining the utility of the extrusion.

A more general consideration would be to include the depth and the width of the profile in the design variable set, or further including entire unit information such as horizontal and stack mullion parameters as well. Moreover, the cost function can be augmented to include parameters such as fabrication and finishing costs in addition to the efficiency index.

7 SUMMARY

In this report the concept of structural optimization was explored and its relevance to façade designs were examined. The potential value of the concept was demonstrated through descriptive examples. The next step in development of this topic is to create simplified algorithms that can be easily incorporated into the course of day-to-day designs. This streamlining approach should remove complex and cumbersome technical obstacles from creating efficient, economical and elegant designs.



Original Design

Design variables

Design (A) (Wt = 2.14 plf)

Design (B) (Wt = 1.76 plf)

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